

MHD convection flow of viscous incompressible fluid over a stretched vertical flat plate in the presence of thermal radiation, viscous dissipation and Hall current effect using similarity solutions and HPM technique

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Abstract:

The effect of thermal radiation, viscous dissipation and hall current of the MHD convection flow of the viscous incompressible fluid over a stretched vertical flat plate has been discussed by using regular perturbation and homotopy perturbation technique with similarity solutions. The influence of various physical parameters on velocity, cross flow velocity and temperature of fluid has been obtained numerically and through graphs.

Key words: Thermal Radiation; Viscous Dissipation; Hall Current effect; MHD flow; Homotopy Perturbation Technique; Stretched flat plate

I. Introduction:

The influence of stretching sheet and the various combinations of additional effects of boundary layer flow problem has many industrial applications such as polymer sheet or filament extrusion from a dye or long thread between feed roll or wind-up roll, glass fiber and paper production, drawing of plastic films, liquid films in condensation process. These highly applicable phenomena in practical problems attract many researchers.

The pioneering studies have been performed by Sakiadis (1961). The study of stretching surfaces and the several combinations of additional effects on the stretching problems are important in many practical applications because the production of sheeting material arises in a number of industrial manufacturing processes and includes both metal and polymer sheets. In the manufacture of the latter, the material is in a molten phase when thrust through an extrusion die and then cools and solidifies some distance away from the die before arriving at the collecting stage. The quality of the resulting sheeting material, as well as the cost of production, is affected by the speed of collection and the heat transfer rate, and a knowledge of the flow properties of the ambient fluid is clearly desirable in Banks and Zaturka (1986) Sparrow and Abraham (2005) pointed the very important practical problem of the thermal processing of sheet-like materials which is a necessary operation in the production of paper, linoleum, polymeric sheets, roofing shingles, insulating materials, fine-fiber mattes. The effects of Hall current and chemical reaction on the hydro magnetic flow of a stretching vertical surface is studied by Salem and Abd El-Aziz (2008). Ghosh

(2009) have studied the Hall effects in a parallel plate channel, while Abd El-Aziz (2010) has analyzed the effects of Hall currents on the flow and heat transfer of an electrically conducting fluid over an unsteady stretching surface in the presence of a strong magnetic field.

Gnanaswara Reddy and Bhaskar Reddy (2011) investigated mass transfer and heat generation effects on MHD free convection flow past an inclined vertical surface in a porous medium. MHD boundary-layer flow over a stretching surface with internal heat generation or absorption was studied by Basiri Parsa (2013). Gnanaswara Reddy (2012) analyzed thermophoresis, viscous dissipation and joule heating effects on steady MHD heat and mass transfer flow over an inclined radiative isothermal permeable surface with variable thermal conductivity. Ali, Nazar and Arifin were consider the effect of Hall current on MHD mixed convection boundary layer flow over a stretched vertical flat plate. Recently Gnanaswara Reddy (2014) study the Influence of thermal radiation, viscous dissipation and Hall current on MHD convection flow over a stretched vertical flat plate.

The aim of the present paper is to investigate the influence of thermal radiation, viscous dissipation and hall current on steady MHD mixed convection boundary layer flow over a stretched vertical flat plate. The non linear coupled partial differential equations are reduced using similarity solutions and further these are solved by Homotopy Perturbation Method. The effects of various governing parameters on the velocity, cross flow velocity, temperature, skin-friction coefficient and Nusselt number are own in figures and tables and discussed further in detail.

II. Homotopy Perturbation Method:

The Homotopy Perturbation Method is a combination of classical Perturbation Technique and Homotopy Theory, which has eliminated the limitations of the traditional perturbation methods. A brief introduction of Homotopy Perturbation Method is given below:

$$L(u) + N(u) - f(r) = 0, r \in \Omega \quad \dots (a)$$

with boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right) = 0, r \in \Gamma \quad \dots (b)$$

here L is the linear operator, N is Nonlinear operator, B is boundary operator and $f(r)$ is known analytic function and Γ is the boundary of the domain Ω .

A Homotopy $v(r, p): \Omega \times [0,1] \rightarrow \mathbb{R}$ for the problem mentioned in equation (a) is

$$H(v, p) = (1 - p)[L(v) - L(v_0)] + p[L(v) + N(v) - f(r)] = 0 \quad \dots (c)$$

or

$$H(v, p) = L(v) - L(v_0) + p[L(v_0) + N(v) - f(r)] = 0 \quad \dots (d)$$

where $p \in [0,1]$ is an embedding parameter and v_0 is an initial approximation of equation (a) which satisfies boundary conditions. It follows from equation (c) and equation (d) that

$$H(v, 0) = L(v) - L(v_0) \text{ and } H(v, 1) = L(v) + N(v) - f(r) \quad \dots (e)$$

The changing process of p from zero to unity is just that of $v(r, p)$ from $v_0(r)$ to $v(r)$. In topology, this is called deformation and $L(v) - L(v_0)$ and $L(v) + N(v) - f(r)$ are called homotopic in topology.

Let

$$v = v_0 + pv_1 + p^2v_2 + \dots \quad \dots (f)$$

And setting $p = 1$ result in an approximate solution of equation (a)

$$u = \lim_{p \rightarrow 1} v = v_0 + v_1 + v_2 + \dots \quad \dots (g)$$

The series of equation (g) is convergent for most of the cases. However, the convergent rate is depends upon the nonlinear operator $N(v)$, the following options are already suggested by He (1999):

1. The second derivative of $N(v)$ with respect to v must be small because the parameter may be relatively large i.e. $p \rightarrow 1$.
2. The norm of $L^{-1}\left(\frac{\partial N}{\partial u}\right)$ must be smaller than one so that the series is convergent.

III. Mathematical Analysis:

Consider the steady incompressible mixed convection flow of a viscous electrically-conducting fluid past a three dimensional uniformly stretched flat plate in the vertical direction. The stationary frame of reference (x, y, z) is chosen such that velocity is proportional to the distance from the fixed origin O also the x -axis is along the direction of motion of the surface, the y -axis is normal to the surface and the z -axis is transverse to the xy -plane. The external magnetic field is assumed to be constant H_0 and is applied in the positive y -direction also the sheet has a variable temperature $T_w(x)$ at the surface while T_∞ is the free stream temperature yield that $T_w(x) > T_\infty$ corresponds to a heated plate and

$T_w(x) < T_\infty$ corresponds to a cooled plate.

It is assumed that the electron pressure gradient, the ion slip and the thermo-electric effects are neglected and in influence of Hall effects the generalized Ohm's law can be written as

$$j = \sigma(E + \mu_e v * H - \frac{\mu_e}{en_e} j * H) \quad \dots (1)$$

Where μ_e and e stand for the electron number density and the electric charge, respectively and the electrical conductivity, σ is given by

$$\sigma = \frac{e^2 n_e T_e}{m_e} \quad \dots (2)$$

Where T_e and m_e are the electron collision time and the mass of an electron, respectively. The effect of Hall current gives rise to a force in the z-direction resulting in a cross-flow in this direction and thus the flow becomes three-dimensional. Using the boundary layer variables it is observed that the physical variable does not depends on the z -coordinate.

Under the above mentioned assumptions and usual Boussinesque approximation, the governing equations for the relevant fluid flow are given as:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \dots (3)$$

Momentum equations:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma B_0^2}{\rho(1+m^2)}(u + mw) \quad \dots (4)$$

$$u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} = \nu \frac{\partial^2 w}{\partial y^2} + \frac{\sigma B_0^2}{\rho(1+m^2)}(mu - w) \quad \dots (5)$$

Energy Equation:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad \dots (6)$$

With the boundary conditions

$$u = u_w(x), v = 0, w = 0, T = T_w(x) \text{ at } y = 0$$

$$u \rightarrow 0, w \rightarrow 0, T \rightarrow T_\infty \text{ as } y \rightarrow \infty \quad \dots (7)$$

Using the Rosseland approximation, the radiative heat flux q_r is given by

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y} \quad \dots (8)$$

Where σ_s is the Stefan–Boltzmann constant and k_e is the mean absorption coefficient. Considering Rosseland approximation, the present analysis is limited to optically thick fluids. If the temperature difference within the flow is sufficiently small, then Eq (8) can be linearized by expanding T^4 using the Taylor series about T_∞ , neglecting higher order terms as

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad \dots (9)$$

Using Eq (8) and Eq (9) the Eq (6) reduces to

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha(1 + R) \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left[\left(\frac{\partial u}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial y} \right)^2 \right] \quad \dots (10)$$

Where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity and $R = 16\sigma^* \frac{T_\infty^3}{3kk^*}$ is the radiation parameter.

It is assumed that $u_w(x)$ and $T_w(x)$ are varies linearly with variable x as $u_w(x) = cx, (c > 0)$ and $T_w(x) = T_\infty + ax$... (11)

where c and a are constants.

Noted that

For $a > 0, (T_w(x) > T_\infty)$, the plate is heated

and for $a < 0, (T_w(x) < T_\infty)$, the plate is cooled.

IV. Method of solution:

We introduce the similarity solutions for Eq(3), Eq(4), Eq(5) and Eq(10) in the form of

$$u = cx f'(\eta), v = -(cv)^{1/2} f(\eta), w = cx h(\eta), \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

$$\eta = \left(\frac{c}{\nu} \right)^{1/2} y, M = \frac{\sigma B_0^2}{c\rho}, Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^2}, Re_x = \frac{u_w(x)}{\nu},$$

$$\lambda = \frac{Gr_x}{Re_x^2}, Pr = \frac{\nu}{\alpha}, Ec = \frac{\nu^2}{c_p(T_w - T_\infty)} \quad \dots (12)$$

Where Pr is the Prandtl Number, M is the magnetic parameter, m is the hall parameter, λ is the constant of buoyancy or mixed convection parameter, Gr_x is the local Grashof Number, Re_x is the local Reynold Number and Ec is the Eckert Number. Substituting these assumptions in Eq (4), Eq(5) and Eq(10), we get

$$f''' + ff'' - f'^2 - \frac{M}{1+m^2}(f' + mh) + \lambda\theta = 0 \quad \dots (13)$$

$$h'' + fh' - f'h + \frac{M}{1+m^2}(mf' - h) = 0 \quad \dots (14)$$

$$(1 + R)\theta'' + Pr [f\theta' - f'\theta + Ec(f''^2 + h''^2)] = 0 \quad \dots (15)$$

The corresponding boundary conditions are

$$\begin{aligned} \text{at } \eta = 0 : f = 0, f' = 1, h = 0, \theta = 1 \\ \text{as } \eta \rightarrow \infty : f' \rightarrow 0, h \rightarrow 0, \theta \rightarrow 0 \end{aligned} \quad \dots (16)$$

The Homotopy for the above three equations are following:

$$H(f, p) = (1-p) \left[f''' - \frac{M}{1+m^2} f' - \left(1 - \frac{M}{1+m^2}\right) e^{-\eta} \right] + p \left[f''' + ff'' - f'^2 - \frac{M}{1+m^2} (f' + mh) + \lambda \theta \right] = 0 \quad \dots (17)$$

$$H(h, p) = (1-p) \left[h'' - \frac{M}{1+m^2} h - \left(1 - \frac{M}{1+m^2}\right) e^{-\eta} + \left(4 - \frac{M}{1+m^2}\right) e^{-2\eta} \right] + p \left[h'' + fh' - f'h + \frac{M}{1+m^2} (mf' - h=0) \right] \quad \dots (18)$$

$$H(\theta, p) = (1-p) [(1+R)\theta'' - (1+R)e^{-\eta}] + p \left[(1+R)\theta'' + Pr \left\{ f\theta' - f'\theta + Ec (f'^2 + h'^2) \right\} \right] = 0 \quad \dots (19)$$

Let

$$\begin{aligned} f &= f_0 + pf_1 + p^2 f_2 + \dots \\ h &= h_0 + ph_1 + p^2 h_2 + \dots \\ \theta &= \theta_0 + p\theta_1 + p^2 \theta_2 + \dots \end{aligned} \quad \dots (20)$$

Substituting these assumptions in Eq(17), Eq(18) and Eq(19) and comparing the coefficient of like powers of p, we get

$$p^0 : f_0''' - \frac{M}{1+m^2} f_0' - \left(1 - \frac{M}{1+m^2}\right) e^{-\eta} = 0 \quad \dots (21)$$

$$p^1 : f_1''' - \frac{M}{1+m^2} f_1' + \left(1 - \frac{M}{1+m^2}\right) e^{-\eta} + f_0 f_0'' - f_0'^2 - \frac{M}{1+m^2} m h_0 + \lambda \theta_0 = 0 \quad \dots (22)$$

$$p^0 : h_0'' - \frac{M}{1+m^2} h_0 - \left(1 - \frac{M}{1+m^2}\right) e^{-\eta} + \left(4 - \frac{M}{1+m^2}\right) e^{-2\eta} = 0 \quad \dots (23)$$

$$p^1 : h_1'' - \frac{M}{1+m^2} h_1 + \left(1 - \frac{M}{1+m^2}\right) e^{-\eta} - \left(4 - \frac{M}{1+m^2}\right) e^{-2\eta} + f_0 h_0' - f_0' h_0 + \frac{M}{1+m^2} m f_0' = 0 \dots (24)$$

$$p^0 : (1+R)\theta_0'' - (1+R)e^{-\eta} = 0 \quad \dots (25)$$

$$p^1 : (1+R)\theta_1'' + (1+R)e^{-\eta} + Pr \left\{ f_0 \theta_0' - f_0' \theta_0 + Ec (f_0'^2 + h_0'^2) \right\} = 0 \quad \dots (26)$$

Now the corresponding boundary conditions are

$$\begin{aligned} \text{at } \eta = 0 : f_0 = 0, f_1 = 0, \dots, f_0' = 1, f_1' = 0, \dots, h_0 = 0, h_1 = 0, \dots, \theta_0 = 1, \theta_1 = 0, \dots \\ \text{as } \eta \rightarrow \infty : f_0' \rightarrow 0, f_1' \rightarrow 0, \dots, h_0 \rightarrow 0, h_1 \rightarrow 0, \dots, \theta_0 \rightarrow 0, \theta_1 \rightarrow 0, \dots \end{aligned} \quad \dots (27)$$

The solutions of the Eq(21) to Eq(26) under the corresponding boundary conditions Eq(27) are

$$f_0 = 1 - e^{-\eta} \quad \dots (28)$$

$$h_0 = e^{-\eta} - e^{-2\eta} \quad \dots (29)$$

$$\theta_0 = e^{-\eta} \quad \dots (30)$$

$$f_1 =$$

$$\begin{aligned} - \left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \sqrt{\frac{M}{1+m^2}} \left(\sqrt{\frac{M}{1+m^2}} + 1 \right) + \lambda m / 2 \sqrt{\frac{M}{1+m^2}} \left(2 + \sqrt{\frac{M}{1+m^2}} \right) + \left[\left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \sqrt{\frac{M}{1+m^2}} \left(1 - \right. \right. \\ \left. \left. M1+m2 - \lambda m / 2 M1+m24 - M1+m2e - M1+m2\eta - M1+m2 + Mm1+m2 - \lambda 1 - M1+m2e - \eta + Mm1+m224 - M \right. \right. \\ \left. \left. 1+m2e - 2\eta \right] \quad \dots (31) \end{aligned}$$

$$h_1 =$$

$$\begin{aligned} \frac{M(1+m)}{1+m^2} / \left(1 - \frac{M}{1+m^2} \right) e^{-\eta} - \left(6 + \frac{2Mm}{1+m^2} - \frac{M}{1+m^2} \right) / \left(4 - \frac{M}{1+m^2} \right) e^{-2\eta} + \left[- \frac{M(1+m)}{1+m^2} / \left(1 - \frac{M}{1+m^2} \right) + \right. \\ \left. 6 + 2Mm1+m2 - M1+m24 - M1+m2e - M1+m2\eta \right] \quad \dots (32) \end{aligned}$$

$$\theta_1 = -(1+R)e^{-\eta} + Pre^{-\eta} - Pr Ec \left(\frac{1}{2} e^{-2\eta} - \frac{4}{9} e^{-3\eta} + \frac{1}{4} e^{-4\eta} \right) \quad \dots (33)$$

Where $(1+R) = Pr(1 + Ec 11/36)$

The values f, h and θ are obtained as given below

$$f = \lim_{p \rightarrow 1} f = f_0 + f_1 + f_2 + \dots$$

$$h = \lim_{p \rightarrow 1} h = h_0 + h_1 + h_2 + \dots$$

$$\theta = \lim_{p \rightarrow 1} \theta = \theta_0 + \theta_1 + \theta_2 + \dots$$

Now

$$f = 1 - e^{-\eta} - \left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \sqrt{\frac{M}{1+m^2}} \left(\sqrt{\frac{M}{1+m^2}} + 1 \right) + \frac{\lambda m}{2\sqrt{\frac{M}{1+m^2}} \left(2 + \sqrt{\frac{M}{1+m^2}} \right)} + \left[\left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \sqrt{\frac{M}{1+m^2}} \left(1 - \frac{M}{1+m^2} \right) - \frac{\lambda m}{2\sqrt{\frac{M}{1+m^2}} \left(4 - \frac{M}{1+m^2} \right)} \right] e^{-\sqrt{\frac{M}{1+m^2}} \eta} - \left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \left(1 - \frac{M}{1+m^2} \right) e^{-\eta} + \frac{Mm}{1+m^2} / 2 \left(4 - \frac{M}{1+m^2} \right) e^{-2\eta} + \dots \quad \dots (34)$$

$$h = e^{-\eta} - e^{-2\eta} + \left[-\frac{M(1+m)}{1+m^2} / \left(1 - \frac{M}{1+m^2} \right) + \left(6 + \frac{2Mm}{1+m^2} - \frac{M}{1+m^2} \right) / \left(4 - \frac{M}{1+m^2} \right) \right] e^{-\sqrt{\frac{M}{1+m^2}} \eta} + \frac{M(1+m)}{1+m^2} / \left(1 - \frac{M}{1+m^2} \right) e^{-\eta} - \left(6 + \frac{2Mm}{1+m^2} - \frac{M}{1+m^2} \right) / \left(4 - \frac{M}{1+m^2} \right) e^{-2\eta} + \dots \quad \dots (35)$$

$$\theta = e^{-\eta} - (1 + R)e^{-\eta} + Pr e^{-\eta} - Pr Ec \left(\frac{1}{2} e^{-2\eta} - \frac{4}{9} e^{-3\eta} + \frac{1}{4} e^{-4\eta} \right) + \dots \quad \dots (36)$$

Hence the velocities for the flow given as

$$u = cx \left[e^{-\eta} - \left[\left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \left(1 - \frac{M}{1+m^2} \right) - \lambda m / \left(4 - \frac{M}{1+m^2} \right) \right] e^{-\sqrt{\frac{M}{1+m^2}} \eta} + M1+m2+Mm1+m2-\lambda1-M1+m2e^{-\eta}-\lambda m/4-M1+m2e^{-2\eta}+\dots \quad \dots (37)$$

$$v = -(cv)^{1/2} \left[1 - e^{-\eta} - \left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \sqrt{\frac{M}{1+m^2}} \left(\sqrt{\frac{M}{1+m^2}} + 1 \right) + \frac{\lambda m}{2\sqrt{\frac{M}{1+m^2}} \left(2 + \sqrt{\frac{M}{1+m^2}} \right)} + M1+m2+Mm1+m2-\lambda M1+m21-M1+m2-\lambda m2M1+m24-M1+m2e^{-M1+m2\eta}-M1+m2+Mm1+m2-\lambda1-M1+m2e^{-\eta}+Mm1+m224-M1+m2e^{-2\eta}+\dots \quad \dots (38)$$

$$w = cx \left[e^{-\eta} - e^{-2\eta} + \left[-\frac{M(1+m)}{1+m^2} / \left(1 - \frac{M}{1+m^2} \right) + \left(6 + \frac{2Mm}{1+m^2} - \frac{M}{1+m^2} \right) / \left(4 - \frac{M}{1+m^2} \right) \right] e^{-\sqrt{\frac{M}{1+m^2}} \eta} + M(1+m)1+m21-M1+m2e^{-\eta}-6+2Mm1+m2-M1+m24-M1+m2e^{-2\eta}+\dots \quad \dots (39)$$

And the temperature is given as

$$T = (T_w - T_\infty) \left[e^{-\eta} - (1 + R)e^{-\eta} + Pr e^{-\eta} - Pr Ec \left(\frac{1}{2} e^{-2\eta} - \frac{4}{9} e^{-3\eta} + \frac{1}{4} e^{-4\eta} \right) + \dots \right] + T_\infty \quad \dots (40)$$

Here $\lambda > 0$ corresponds to the assisting flow (heated plate), also $\lambda < 0$ corresponds to the opposing flow (cooled plate) and $\lambda = 0$ corresponds to the forced convection flow.

V. Skin Friction Coefficient and Heat Transfer Coefficient:

The quantities of physical interest are the coefficient of skin friction C_x and C_z as well as heat transfer coefficient (Nusselt Number) Nu are defined as

$$C_x = \frac{\tau_{wx}^2}{\rho u_w}, \quad C_z = \frac{\tau_{wz}^2}{\rho u_w}, \quad Nu = \frac{xq_w}{k(T_w - T_\infty)} \dots \quad \dots (41)$$

Where k being thermal conductivity of the fluid τ_{w_x} and τ_{w_z} are the shear stresses in the directions of x and z respectively also q_w is the heat flux from the surface of the flat plate are given by

$$\tau_{w_x} = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \tau_{w_z} = \mu \left(\frac{\partial w}{\partial y} \right)_{y=0}, q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

Using eq.(12) and eq.(37) we get

$$C_x Re_x^{1/2} = f''(0), C_z Re_x^{1/2} = h'(0), Nu Re_x^{1/2} = -\theta'(0) \quad \dots (42)$$

Where

$$f''(0) = -1 + \left(\frac{M}{1+m^2} + \frac{Mm}{1+m^2} - \lambda \right) / \left(1 + \sqrt{\frac{M}{1+m^2}} + \left(\frac{Mm}{1+m^2} \right) \right) / \left(2 + \sqrt{\frac{M}{1+m^2}} \right) \quad \dots (43)$$

$$h'(0) = 1 - \frac{M(1+m)}{1+m^2} / \left(1 - \sqrt{\frac{M}{1+m^2}} \right) + \left(6 - \frac{M}{1+m^2} + \frac{2Mm}{1+m^2} \right) / \left(2 - \sqrt{\frac{M}{1+m^2}} \right) \quad \dots (44)$$

$$-\theta'(0) = -R + Pr - \frac{2}{3} Pr Ec \quad \dots (45)$$

VI. Results and discussion:

The Table 1 elucidates the effect of increasing values of hall parameter m and mixed convection parameter λ , the values of $f''(0), h'(0)$ also increases and $f''(0), h'(0)$ increases due to decrement of magnetic parameter M . Table 2 shows that the value of $\theta'(0)$ increase due to increase in radiation parameter R and Eckert Number Ec but this decreases due to increase in Prandtal Number Pr .

It has been observed from the fig. 1 that the velocity increase due to increase in hall parameter m and mixed convection parameter λ also velocity decreases due to increase in magnetic field parameter M . It is observed from the fig.2 that the cross flow velocity increases due to increase in hall parameter m and decrease due to increase in magnetic field parameter M .

It has been observed that the temperature increases due to increase in radiation parameter R and Eckert Number Ec also increases due to decrease in Prandtal Number Pr .

m	M	□	f'(0)	h'(0)
1	1	1	-0.815301	2.613270
1	0.8	1	-0.725536	3.503314
1	1	2	-0.229515	2.613270
2	1	1	-0.560156	3.026459

Table 1: Numerical Values of the skin-friction coefficient

R	Pr	Ec	(- □'(0))
1	0.1	0.01	0.900667
1.1	0.1	0.01	1.000667
1	0.2	0.01	0.801333
1	0.1	0.05	0.903333

Table 2: Numerical values of the heat transfer rate -θ'(0)

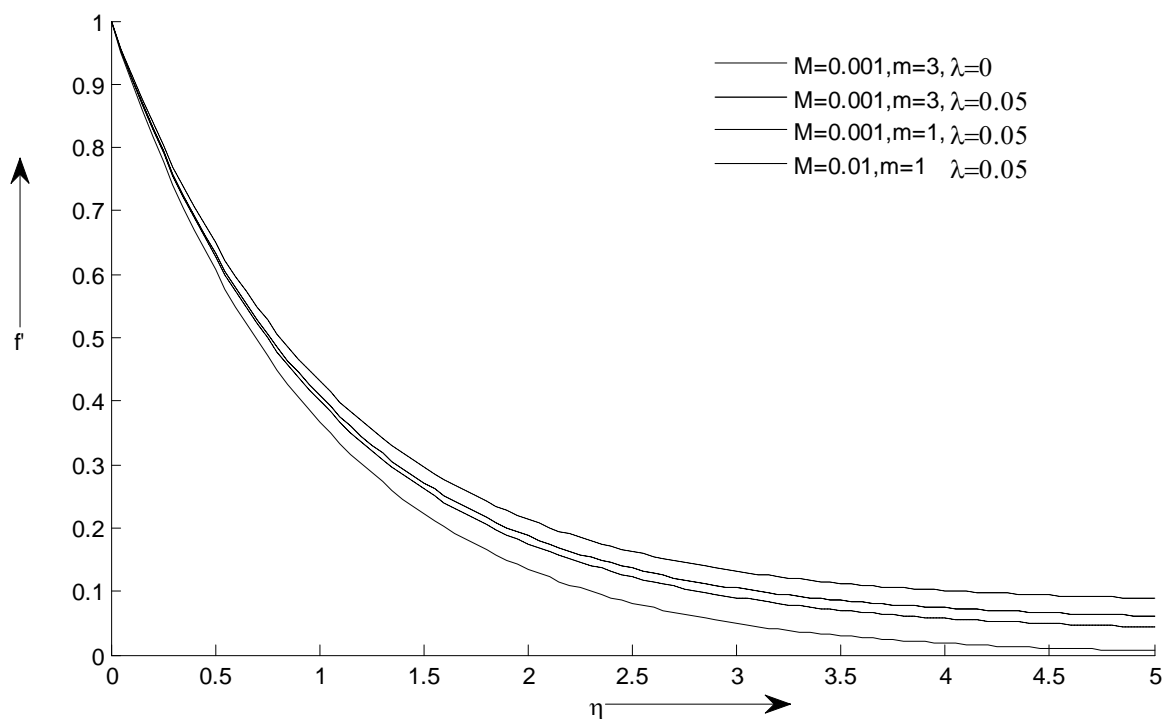


Figure 1: velocity profile for different values of M, m and λ

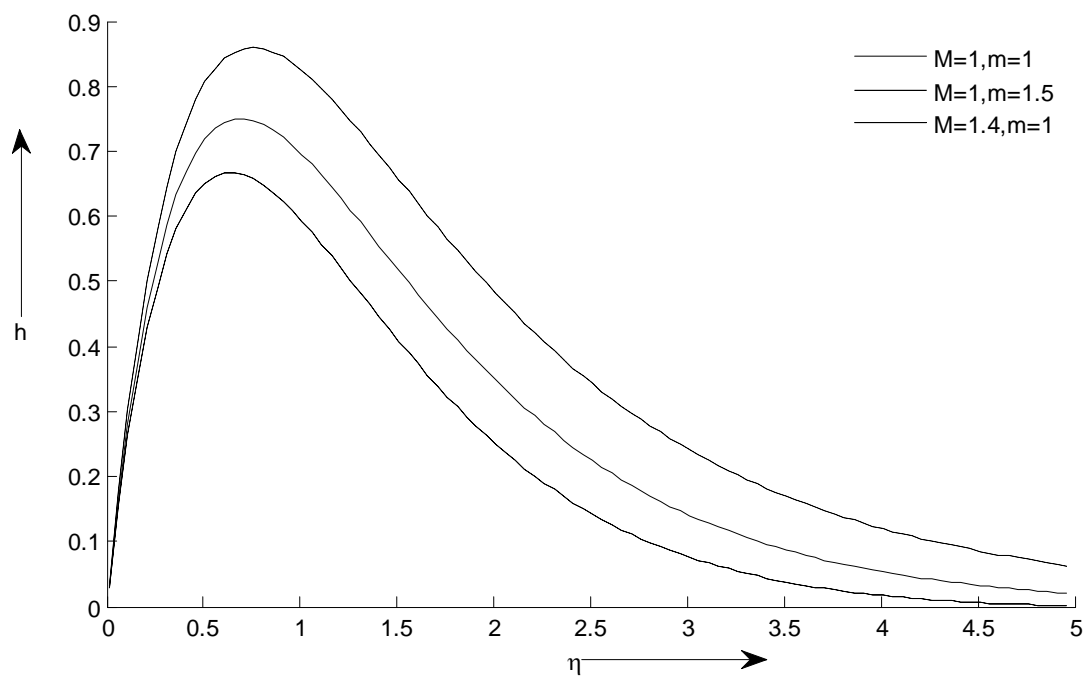


Figure 2: Cross flow velocity profile for different values of M and m

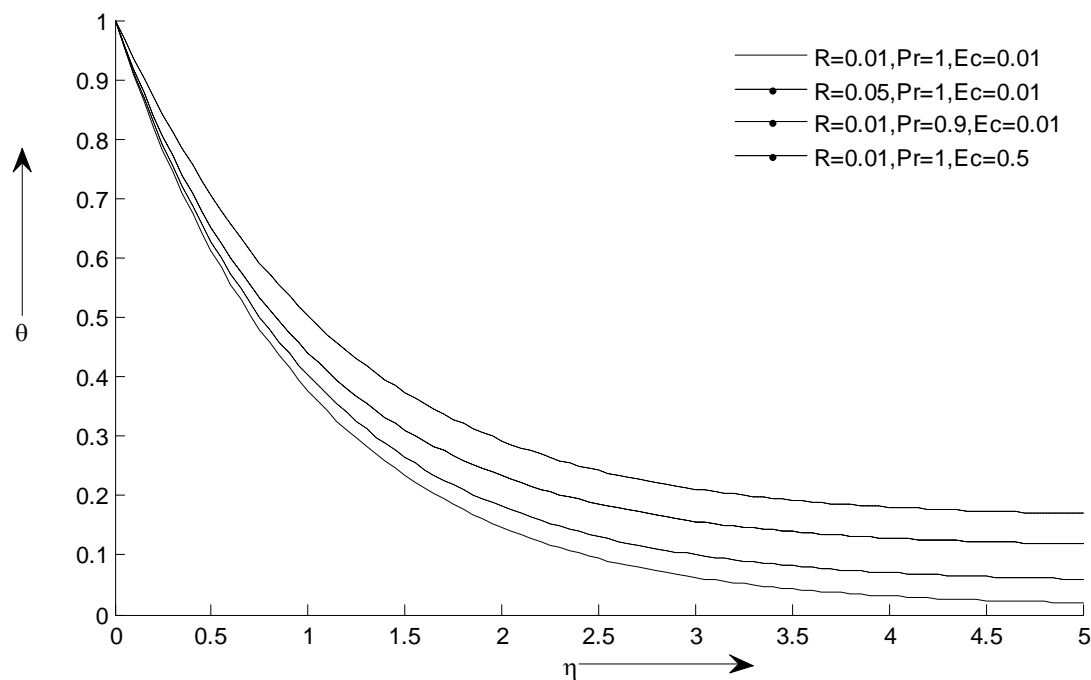


Figure 3: Temperature profiles for different values of R, Pr and Ec

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